Physics of Bowling Balls

David C. Terry
MRC/LA
21 July 2000
Overview

• Summary of game’s objectives
• Important aspects of the game, in order:
  – correct fundamentals (lessons)
  – mental approach (psychology)
  – body control (DNA)
  – equipment (cool high-tech examples)
• Equivalence to golf, horseshoes
Objectives

• Maximize *score* by getting all 10 pins
  – every shot … *repetition*
  – correct impact placement of hit
  – correct entry angle to pins, well-matched with the rotational parameters of delivery
  – ball trajectory well-matched with surface, friction, oil pattern and migration thereof

• If you don’t *strike*, convert every *spare*
Hit Placement & Angle

- **Straight line from gutter to headpin**
  - only 1.5 degree entry angle
  - most hits centered on headpin leave pins on both sides (*split*)
  - want to hit 1-3 pocket (right handers, 86%) or 1-2 pocket (left handers, 14%)

- **Entry angle in 4-6° range is desirable**
Small entry angle

10-pin leave

Restitution= 0.85
Line=30.0 to 25.0
Pocket hit=16.71
Entry angle=1.83

PBA tree blend, 9 units to 5 outside; Purple Hammer (shiny) label shift loaded
Big entry angle

Too much headpin,
this is a bad result.
Perfect Hit

PEA tree blend, 9 units to 5 outside; Purple Hammer (shiny) label shift loaded
Near-Perfect Hit

“Solid” 9 pin

Solid 9 -- Swing 15 to 10; Breakpoint 2; 18 mph; 20 revs; 9 degree entry
Oil Pattern on Lane Surface

Threshold of friction: 3-10 units

5-to-5 Wall: Oil Density at five Distances

- Crown 28-37 ft
- Heads 15-22 ft
Ball Trajectory

- Must *hook* the ball in order to create the large entry angles necessary to score
- Hook is created by:
  - friction between ball and oil/lane surface
  - rotational kinematics of ball, including the *(non-diagonal)* moment of inertia tensor
Rotational Kinematics

- Earth-Centered-Inertial (ECI) and Ball-Centered-Rotational (BCR) coordinate systems
- Pre-1990 bowling:
  - $I$ (moment of Inertia) is diagonal
  - $\hat{\omega}$ (rotational axis vector) is constant in BCR
  - equations of motion relatively simple
- Post-1990 bowling:
  - all three of these have changed ...
Capitalism at Work!

High Performance Bowling Ball Price Trend

Average Retail Price, Dollars

Year

Capitalism gone Wrong?
Mathematics

• ECI-to-BCR is described by orthogonal 3x3 rotation matrix:
  \[ \vec{p}_{ECI} = Q(t) \vec{q}_{BCR} \]
  
• Rotational axis defined by:
  \[ Q \omega_B = 0 \text{ (in BCR)} \]
  \[ Q^t \omega_E = 0 \text{ (in ECR)} \]

• Because Q is orthogonal, \( \dot{Q} Q^t \) is antisymmetric
  \[ \text{one zero and two imaginary eigenvalues} \]
  \[ \omega_E: \text{eigenvector corresponding to null eigenvalue} \]
Friction

- Linear model works for “older” balls
  - $\mathbf{F}_f = -\mu (M \mathbf{g}) \mathbf{V}_c$ friction at contact point
  - $\mathbf{V}_c = \mathbf{V} - a \mathbf{QQ}^T \mathbf{z}$ velocity at contact point

- Post-1991 balls, “reactive resin,” must be modeled with non-linear $\mathbf{F}_f(\mathbf{V}_c)$
  - thermoset elastomers with liquid polymer suspended is surface (bleed when hot)
Ball Track “Flare”

Positive Axis Point:
Intersection of $\omega_B$ with ball’s surface; initial value defined by drilling of grip holes relative to bowler’s wrist position at release.

- **Flare** = Distance track moves across ball, can be 1/2 the ball or more!